

Vertex operators for production of baryon resonances in nucleon-gamma collisions with preserving gauge invariance and analyticity

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Vertex operators for photo- and electro-production of baryon states with arbitrary spin-parity, $\gamma + N \rightarrow B(J^P)$, are constructed. The operators obey gauge invariance and analyticity constraints. Analyticity is realized as a requirement of the generalized Siegert theorem for vertex form factors.

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1. Introduction

The photo- and electro-production of mesons off nucleons provide a direct way for the identification of baryon resonances, that was at the times of the discovery of low-lying states [1–7] as well as at recent studies [8–16].

The present work is devoted to the baryon-nucleon-gamma vertex, $B_{JP}(p) \rightarrow N_{\frac{1}{2}+}(k) + \gamma_{1-}(q)$, where p, k, q are correspondingly four-momenta of baryon state with spin-parity J^P , nucleon and gamma; energy-momentum conservation reads $p = k + q$. Our consideration is based on results of ref. [17] where the case of real photon ($q^2 = 0$) was considered for any J^P . An advantage of the used classification of operators is the fixation not only spin-parity but helicity amplitudes as well that is convenient in data fit procedure.

The case of arbitrary q^2 is the subject of this paper. A problem in the construction of the gauge invariant spin-momentum operators appears because of the pole singularity ($1/q^2$) inherent to the spin part of a vector particle propagator. We eliminate the singularity by adding the longitudinal spin operator, which is nilpotent at $q^2 = 0$, as a cancellation term. Nilpotent operators for improving the analytical structure of operators were used for meson states in [18–20] (see also [21, 22]). The new operators impose a requirement for the decay form factors which is known for real photons as the Siegert theorem [23].

The present consideration is based on spin-operator expansion technique presented in detail in [21, 22], necessary elements of this technique are given in Appendices A, B.

2. Vertices and operators

According to the classification used in [17] we have two types of vertices:

- (i) the (+)-vertices for transitions $B(J^P) \rightarrow N(\frac{1}{2}^+)\gamma$ with spin-parity of the decaying states equal to $J^P = \frac{1}{2}^-, \frac{3}{2}^+, \frac{5}{2}^-, \frac{7}{2}^+, \dots$ and
- (ii) the (-)-vertices for decaying states with $J^P = \frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+, \frac{7}{2}^-, \dots$

2.0.1. The (+)-vertices

Vertices of the (+)-states ($J^P = \frac{1}{2}^-, \frac{3}{2}^+, \frac{5}{2}^-, \dots$) read:

$$\begin{aligned} G_{\alpha_1 \dots \alpha_n}^{(a+)}(k_\perp) &= G_{n+\frac{1}{2}}^{(a+)}(s, q^2) V_{\alpha_1 \dots \alpha_n}^{(a+)\mu}(k_\perp) \epsilon_\mu^{\perp q}, \quad a = 1, 2, 3, \\ V_{\alpha_1 \dots \alpha_n}^{(1+)\mu}(k_\perp) &= i \gamma_\mu^{\perp p} \gamma_5 X_{\alpha_1 \dots \alpha_n}^{(n)}(k_\perp), \\ V_{\alpha_1 \dots \alpha_n}^{(2+)\mu}(k_\perp) &= i \gamma_\nu^{\perp p} \gamma_5 X_{\mu\nu\alpha_1 \dots \alpha_n}^{(n+2)}(k_\perp), \\ V_{\alpha_1 \dots \alpha_n}^{(3+)\mu}(k_\perp) &= i \gamma_\nu^{\perp p} \gamma_5 X_{\nu\alpha_1 \dots \alpha_{n-1}}^{(n)}(k_\perp) g_{\alpha_n\mu}^{\perp p}. \end{aligned} \quad (1)$$

Here $n = J - \frac{1}{2}$ and

$$\begin{aligned} s &= p^2 = (k + q)^2, \quad (qp) = \frac{1}{2}(p^2 - M_N^2 + q^2) \\ k &= k_\perp + p \frac{(kp)}{p^2}, \quad q = -k_\perp + p \frac{(qp)}{p^2}, \\ \gamma_\mu^{\perp p} &= \gamma_{\mu'} g_{\mu'\mu}^{\perp p} = \gamma_{\mu'} (g_{\mu'\mu} - \frac{p_{\mu'} p_\mu}{p^2}). \end{aligned} \quad (2)$$

The form factors $G_{n+\frac{1}{2}}^{(a+)}(s, q^2)$ depend on two momenta squared and the nucleon mass: s, q^2 and $k^2 = M_N^2$. Orbital momentum expansion functions, $X_{\alpha_1 \dots \alpha_n}^{(n)}(k_\perp)$ are presented in Appendix A. Polarization vectors of gamma obey the constraint for vector particles: $\epsilon_\mu q_\mu = 0$.

2.0.2. The (-)-vertices

Vertices of the (-)-states ($J^P = \frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+, \dots$) read:

$$\begin{aligned} G_{\alpha_1 \dots \alpha_n}^{(a-)}(k_\perp) &= G_{n+\frac{1}{2}}^{(a-)}(s, q^2) V_{\alpha_1 \dots \alpha_n}^{(a-)\mu}(k_\perp) \epsilon_\mu^{\perp q}, \quad a = 1, 2, 3 \\ V_{\alpha_1 \dots \alpha_n}^{(1-)\mu}(k_\perp) &= \gamma_\nu^{\perp p} \gamma_\mu X_{\nu\alpha_1 \dots \alpha_n}^{(n+1)}(k_\perp), \\ V_{\alpha_1 \dots \alpha_n}^{(2-)\mu}(k_\perp) &= X_{\mu\alpha_1 \dots \alpha_n}^{(n+1)}(k_\perp), \\ V_{\alpha_1 \dots \alpha_n}^{(3-)\mu}(k_\perp) &= X_{\alpha_2 \dots \alpha_L}^{(n-1)}(k_\perp) g_{\alpha_1\mu}^{\perp p} \end{aligned} \quad (3)$$

2.1. Operators at $q^2 \neq 0$

Equations (1),(3) are written for the convolution of operators with photon polarization vectors. To activate the photon polarization indices at $q^2 \neq 0$ one can use

the completeness conditions for polarization vectors which are written as follows:

$$\sum_a \epsilon_{\mu'}^{\perp q}(a) \epsilon_{\mu}^{\perp q*}(a) = g_{\mu'\mu} - \frac{q_{\mu'} q_{\mu}}{q^2} \quad (4)$$

Therefore we write for operators at $q^2 \neq 0$:

$$G_{\alpha_1 \dots \alpha_n}^{(a\pm)\mu}(k_{\perp}) = G_{n+\frac{1}{2}}^{(a\pm)}(s, q^2) V_{\alpha_1 \dots \alpha_n}^{(a\pm)\mu'}(k_{\perp}) (g_{\mu'\mu} - \frac{q_{\mu'} q_{\mu}}{q^2}), \quad a = 1, 2, 3 \quad (5)$$

The operators (5) are appropriate for working at $q^2 \neq 0$, for example, with massive vector particles, ($q^2 = M^2$).

The pole singularity ($1/q^2$) can be eliminated, if it is necessary, by the form factor constraint: $G_{n+\frac{1}{2}}^{(a\pm)}(s, q^2 = 0) = 0$. But in some states it turns out to amusing results. Another way to eliminate the ($1/q^2$)-pole is related to the introduction of the longitudinal component.

3. Longitudinal component and generalized Siegert theorem

The elimination of the pole singularity can be performed by adding to the operator ($g_{\mu'\mu} - \frac{q_{\mu'} q_{\mu}}{q^2}$) a longitudinal component (orthogonal to q) with an appropriate constant A :

$$\begin{aligned} \left(g_{\mu'\mu} - \frac{q_{\mu'} q_{\mu}}{q^2} \right) &\rightarrow \left[g_{\mu'\mu} - \frac{q_{\mu'} q_{\mu}}{q^2} + A q_{\mu'} \left(q_{\mu} - \frac{q^2}{(pq)} p_{\mu} \right) \right]_{A=1/q^2} \\ &= \left[g_{\mu'\mu} - \frac{q_{\mu'} q_{\mu}}{q^2} + \frac{1}{q^2} q_{\mu'} \left(q_{\mu} - \frac{q^2}{(pq)} p_{\mu} \right) \right] = \left(g_{\mu'\mu} - \frac{q_{\mu'} p_{\mu}}{(pq)} \right) \end{aligned} \quad (6)$$

The singularity $1/q^2$ is cancelled but we face a new singularity at $(pq) = 0$, this one can be eliminated by a zero form factor. But the particularity consists in the fact that the $1/(pq)$ pole is related to the longitudinal component, therefore one can construct two transverse operators without the $1/(pq)$ pole singularities.

Correspondingly, one vertex form factor should satisfy the generalized Siegert theorem, we nominate this operator (and form factor) as $a = 3$:

$$\left[G_J^{(3\pm)}(s, q^2) \right]_{s=M_N^2 - q^2} = 0, \quad a = 3. \quad (7)$$

Also, let us emphasize that the longitudinal operator $L_{\mu} = (q_{\mu} - \frac{q^2}{(pq)} p_{\mu})$ which is attended at (6) is nilpotent at $q^2 = 0$: ($L_{\mu} L_{\mu} = 0$). The nilpotent operators were used in [18–20] for the description of photon interactions with mesons.

3.1. Separation of the longitudinal operators and analyticity constraint

Separation of the longitudinal operators is performed here in detail.

3.1.1. The operators for the $(-)$ -states

To perform the separation of the longitudinal and transverse operators let us fix the $V_{\alpha_1 \dots \alpha_n}^{G(3-)\mu}(k_\perp)$ as a basic one:

$$\begin{aligned} V_{\alpha_1 \dots \alpha_n}^{G(3-)\mu}(k_\perp) &\equiv \mathfrak{V}_{\alpha_1 \dots \alpha_n}^{(3-)\mu}(k_\perp) = X_{\alpha_2 \dots \alpha_n}^{(n-1)}(k_\perp) g_{\alpha_1 \mu'}^\perp (g_{\mu' \mu} - \frac{q_{\mu'} p_\mu}{(pq)}) \\ &= X_{\alpha_2 \dots \alpha_n}^{(n-1)}(k_\perp) (g_{\alpha_1 \mu} - \frac{q_{\alpha_1} p_\mu}{(pq)}) \end{aligned} \quad (8)$$

The pole singularity at $(pq) = 0$ should be eliminated by requirement (7), that is the generalized Siegert theorem.

Let us now consider the vertex $V_{\alpha_1 \dots \alpha_n}^{G(2-)\mu}(k_\perp)$:

$$\begin{aligned} V_{\alpha_1 \dots \alpha_n}^{G(2-)\mu}(k_\perp) &= X_{\mu' \alpha_1 \dots \alpha_n}^{(n+1)}(k_\perp) (g_{\mu' \mu} - \frac{q_{\mu'} p_\mu}{(pq)}) \\ &= \left(X_{\mu \alpha_1 \dots \alpha_n}^{(n+1)}(k_\perp) + X_{\mu' \alpha_1 \dots \alpha_n}^{(n+1)}(k_\perp) \frac{k_\mu^\perp p_\mu}{(pq)} \right) \\ &= \left(X_{\mu \alpha_1 \dots \alpha_n}^{(n+1)}(k_\perp) - k_\perp^2 \frac{2n-1}{n} X_{\alpha_2 \dots \alpha_n}^{(n-1)}(k_\perp) g_{\alpha_1 \mu} \right. \\ &\quad \left. + k_\perp^2 \frac{2n-1}{n} X_{\alpha_2 \dots \alpha_n}^{(n-1)}(k_\perp) \left(g_{\alpha_1 \mu} - \frac{q_{\alpha_1} p_\mu}{(pq)} \right) \right). \end{aligned} \quad (9)$$

The last term is proportional to $V_{\alpha_1 \dots \alpha_n}^{G(3-)\mu}(k_\perp)$ and can be removed. Recall $k_\perp = -q + p \frac{(pq)}{p^2}$ and $k_\perp^2 = q^2 - \frac{(pq)^2}{p^2}$. Also, we have used:

$$X_{\mu' \alpha_1 \dots \alpha_n}^{(n+1)}(k_\perp) \frac{k_\mu^\perp p_\mu}{(pq)} = k_\perp^2 \frac{2n-1}{n} X_{\alpha_2 \dots \alpha_n}^{(n-1)}(k_\perp) \left(-\frac{q_{\alpha_1} p_\mu}{(pq)} \right). \quad (10)$$

We see that the singular term can be removed by redefinition of the vertex:

$$\begin{aligned} V_{\alpha_1 \dots \alpha_n}^{G(2-)\mu}(k_\perp) &\rightarrow \mathfrak{V}_{\alpha_1 \dots \alpha_n}^{(2-)\mu}(k_\perp) \\ &= V_{\alpha_1 \dots \alpha_n}^{G(2-)\mu}(k_\perp) - q^2 \frac{2n-1}{n} V_{\alpha_1 \dots \alpha_n}^{G(3-)\mu}(k_\perp) \end{aligned} \quad (11)$$

The redefined vertex $\mathfrak{V}_{\alpha_1 \dots \alpha_n}^{(2-)\mu}(k_\perp)$ is analytical at $(pq) = 0$ and $q_\mu \mathfrak{V}_{\alpha_1 \dots \alpha_n}^{(2-)\mu}(k_\perp) = 0$.

Redefinition of the $V_{\alpha_1 \dots \alpha_n}^{G(1-)\mu}(k_\perp)$ looks as follows:

$$\begin{aligned} V_{\alpha_1 \dots \alpha_n}^{G(1-)\mu}(k_\perp) &= \gamma_\nu^\perp \gamma_\mu X_{\nu \alpha_1 \dots \alpha_n}^{(n+1)}(k_\perp) - \gamma_\nu^\perp \hat{q} X_{\nu \alpha_1 \dots \alpha_n}^{(n+1)}(k_\perp) \frac{p_\mu}{(pq)} \\ &= \gamma_\nu^\perp \gamma_\mu X_{\nu \alpha_1 \dots \alpha_n}^{(n+1)}(k_\perp) - \gamma_\nu^\perp \hat{q} k_\nu^\perp \frac{2n+1}{n+1} X_{\alpha_1 \dots \alpha_n}^{(n)}(k_\perp) \frac{p_\mu}{(pq)} \\ &= \gamma_\nu^\perp \gamma_\mu X_{\nu \alpha_1 \dots \alpha_n}^{(n+1)}(k_\perp) - \hat{k}_\perp \hat{q} \frac{2n+1}{n+1} \cdot \frac{2n-1}{n} k_\alpha^\perp X_{\alpha_2 \dots \alpha_n}^{(n)}(k_\perp) \frac{p_\mu}{(pq)} \end{aligned} \quad (12)$$

and

$$\hat{k}_\perp \hat{q} = (pq) - \frac{(pq)}{p^2} \hat{p} \hat{k} - q^2 \quad (13)$$

To cancel the pole item in (12), $\sim 1/(pq)$, we add $C\mathfrak{V}_{\alpha_1\ldots\alpha_n}^{(3-)\mu}(k_\perp)$ with proper coefficient C :

$$C = -q^2 \frac{4n^2 - 1}{n(n+1)}. \quad (14)$$

The redefined operator reads:

$$\mathfrak{V}_{\alpha_1\ldots\alpha_n}^{(1-)\mu}(k_\perp) = V_{\alpha_1\ldots\alpha_n}^{G(1-)\mu}(k_\perp) - q^2 \frac{4n^2 - 1}{n(n+1)} V_{\alpha_1\ldots\alpha_n}^{G(3-)\mu}(k_\perp) \quad (15)$$

3.1.2. The operators for the (+)-states

We have three operators:

$$\begin{aligned} V_{\alpha_1\ldots\alpha_n}^{G(1+)\mu}(k_\perp) &= i\gamma_{\mu'}^{\perp p} \gamma_5 X_{\alpha_1\ldots\alpha_n}^{(n)}(k_\perp) (g_{\mu'\mu} - \frac{q_{\mu'} p_\mu}{(pq)}), \\ V_{\alpha_1\ldots\alpha_n}^{G(2+)\mu}(k_\perp) &= i\gamma_\nu^{\perp p} \gamma_5 X_{\mu'\nu\alpha_1\ldots\alpha_n}^{(n+2)}(k_\perp) (g_{\mu'\mu} - \frac{q_{\mu'} p_\mu}{(pq)}), \\ V_{\alpha_1\ldots\alpha_n}^{G(3+)\mu}(k_\perp) &= i\gamma_\nu^{\perp p} \gamma_5 X_{\nu\alpha_1\ldots\alpha_{n-1}}^{(n)}(k_\perp) g_{\alpha_n\mu'}^{\perp p} (g_{\mu'\mu} - \frac{q_{\mu'} p_\mu}{(pq)}). \end{aligned} \quad (16)$$

The reorganized operators $V_{\alpha_1\ldots\alpha_n}^{G(a+)\mu}(k_\perp) \rightarrow \mathfrak{V}_{\alpha_1\ldots\alpha_n}^{(a+)\mu}(k_\perp)$ (with $a = 1, 2, 3$) read:

$$\begin{aligned} V_{\alpha_1\ldots\alpha_n}^{G(1+)\mu}(k_\perp) &\rightarrow \mathfrak{V}_{\alpha_1\ldots\alpha_n}^{(1+)\mu}(k_\perp) = V_{\alpha_1\ldots\alpha_n}^{G(1+)\mu}(k_\perp) - V_{\alpha_1\ldots\alpha_n}^{G(3+)\mu}(k_\perp) \\ V_{\alpha_1\ldots\alpha_n}^{G(2+)\mu}(k_\perp) &\rightarrow \mathfrak{V}_{\alpha_1\ldots\alpha_n}^{(2+)\mu}(k_\perp) = V_{\alpha_1\ldots\alpha_n}^{G(2+)\mu}(k_\perp) - q^2 \frac{(2n+1)n}{(2n-1)(n+1)} V_{\alpha_1\ldots\alpha_n}^{G(3+)\mu}(k_\perp) \\ V_{\alpha_1\ldots\alpha_n}^{G(3+)\mu}(k_\perp) &\equiv \mathfrak{V}_{\alpha_1\ldots\alpha_n}^{(3+)\mu}(k_\perp) \\ &= i\gamma_\nu^{\perp p} \gamma_5 \left(X_{\nu\alpha_2\ldots\alpha_n}^{(n)}(k_\perp) g_{\alpha_1\mu}^{\perp p} - X_{\nu\alpha_2\ldots\alpha_n}^{(n)}(k_\perp) q_{\alpha_1} \frac{p_\mu}{(pq)} \right). \end{aligned} \quad (17)$$

We see that all operators are subjects of the gauge requirement: $q_\mu \mathfrak{V}_{\alpha_1\ldots\alpha_n}^{(a\pm)\mu}(k_\perp) = 0$, $a = 1, 2, 3$.

3.2. The generalized Siegert theorem

Operators with taken into account form factors read as follows:

$$\mathfrak{G}_{\alpha_1\ldots\alpha_n}^{(a\pm)\mu}(k_\perp) = G_J^{(a\pm)}(s, q^2) \mathfrak{V}_{\alpha_1\ldots\alpha_n}^{(a+)\mu}(k_\perp), \quad a = 1, 2, 3. \quad (18)$$

The operators $\mathfrak{V}_{\alpha_1\ldots\alpha_n}^{(a+)\mu}(k_\perp)$ are constricted in such a way that the pole singularity $1/(pq)$ attends at $(a = 3)$ -component only. Correspondingly form factors $G^{(3\pm)}(s, q^2)$ obey to generalized Siegert theorem presented in eq.(7): $G^{(3\pm)}(s, q^2) \sim (s - M_N^2 + q^2)$.

4. Different versions for construction of the (\pm) -operators

In the previous Section we constructed $(+)$ -operators ($J^P = \frac{1}{2}^-, \frac{3}{2}^+, \frac{5}{2}^-, \dots$) and $(-)$ -operators ($J^P = \frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+, \dots$) which obey analyticity and gauge invariance constraints that are $\mathfrak{G}_{\alpha_1 \dots \alpha_n}^{(a+)\mu}(k_\perp)$ and $\mathfrak{G}_{\alpha_1 \dots \alpha_n}^{(a-)\mu}(k_\perp)$. One can construct other sets of operators, $\tilde{\mathfrak{G}}_{\alpha_1 \dots \alpha_n}^{(a+)\mu}(k_\perp) = \mathfrak{G}_{\alpha_1 \dots \alpha_n}^{(a-)\mu}(k_\perp) \cdot i\gamma_5$ and $\tilde{\mathfrak{G}}_{\alpha_1 \dots \alpha_n}^{(a-)\mu}(k_\perp) = \mathfrak{G}_{\alpha_1 \dots \alpha_n}^{(a+)\mu}(k_\perp) \cdot i\gamma_5$. We have four equivalent definitions of the (\pm) -operators:

$$\begin{array}{c|c}
 \mathfrak{G}_{\alpha_1 \dots \alpha_n}^{(1+)\mu}(k_\perp) & \mathfrak{G}_{\alpha_1 \dots \alpha_n}^{(1-)\mu}(k_\perp) \\
 \mathfrak{G}_{\alpha_1 \dots \alpha_n}^{(2+)\mu}(k_\perp) & \mathfrak{G}_{\alpha_1 \dots \alpha_n}^{(2-)\mu}(k_\perp) \\
 \mathfrak{G}_{\alpha_1 \dots \alpha_n}^{(3+)\mu}(k_\perp) & \mathfrak{G}_{\alpha_1 \dots \alpha_n}^{(3-)\mu}(k_\perp) \\
 \hline
 \tilde{\mathfrak{G}}_{\alpha_1 \dots \alpha_n}^{(1+)\mu}(k_\perp) = \mathfrak{G}_{\alpha_1 \dots \alpha_n}^{(1-)\mu}(k_\perp) \cdot i\gamma_5 & \tilde{\mathfrak{G}}_{\alpha_1 \dots \alpha_n}^{(1-)\mu}(k_\perp) = \mathfrak{G}_{\alpha_1 \dots \alpha_n}^{(1+)\mu}(k_\perp) \cdot i\gamma_5 \\
 \tilde{\mathfrak{G}}_{\alpha_1 \dots \alpha_n}^{(2+)\mu}(k_\perp) = \mathfrak{G}_{\alpha_1 \dots \alpha_n}^{(2-)\mu}(k_\perp) \cdot i\gamma_5 & \tilde{\mathfrak{G}}_{\alpha_1 \dots \alpha_n}^{(2-)\mu}(k_\perp) = \mathfrak{G}_{\alpha_1 \dots \alpha_n}^{(2+)\mu}(k_\perp) \cdot i\gamma_5 \\
 \tilde{\mathfrak{G}}_{\alpha_1 \dots \alpha_n}^{(3+)\mu}(k_\perp) = \mathfrak{G}_{\alpha_1 \dots \alpha_n}^{(3-)\mu}(k_\perp) \cdot i\gamma_5 & \tilde{\mathfrak{G}}_{\alpha_1 \dots \alpha_n}^{(3-)\mu}(k_\perp) = \mathfrak{G}_{\alpha_1 \dots \alpha_n}^{(3+)\mu}(k_\perp) \cdot i\gamma_5
 \end{array} \quad (19)$$

Obviously, only the $(3\pm)$ -operators obey the Siegert theorem requirement.

5. Conclusion

We construct gauge invariant operators for vertices of baryon resonances decaying into the nucleon-gamma state, $B(J^P) \rightarrow N(\frac{1}{2}^+) + \gamma$. The operators describe transition vertices of resonances with arbitrary spin-parity, J^P , and arbitrary momenta squared of gamma, q^2 . The suggested operator construction presents the generalization of the scheme of ref. [17] for photon vertices, ($q^2 = 0$).

The key point in the construction of the gauge invariant operators valid at all possible q^2 is the introduction of longitudinal spin operator L_μ which is nilpotent at $q^2 = 0$: $[L_\mu L_\mu]_{q^2=0} = 0$. The addition of the nilpotent operator corrects the analytical structure of spin operators. For meson states such a correction was carried out in [18–20] (see also [21, 22]). The procedure for baryon-gamma states is performed here (Section 3): the pole singularity inherent to spin operators of vector particles ($1/q^2$) is eliminated by adding the nilpotent operator L_μ .

Form factors of the transitions $B(J^P) \rightarrow N(\frac{1}{2}^+) + \gamma$ should satisfy the generalized Siegert requirement: $G_J^{(3\pm)}(s, q^2) = 0$ at $s = M_N^2 - q^2$.

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Appendix A: Angular momentum operators for two-meson systems

We use angular momentum operators $X_{\mu_1 \dots \mu_L}^{(L)}(k^\perp)$, $Z_{\mu_1 \dots \mu_L}^\alpha(k^\perp)$ and the projection operator $O_{\nu_1 \dots \nu_L}^{\mu_1 \dots \mu_L}(\perp P)$ (see [18, 21, 22]). Let us recall their definition.

The operators are constructed from the relative momenta k_μ^\perp and tensor $g_{\mu\nu}^\perp$. Both of them are orthogonal to the total momentum of the system:

$$k_\mu^\perp = \frac{1}{2} g_{\mu\nu}^\perp (k_1 - k_2)_\nu = k_{1\nu} g_{\nu\mu}^{\perp P} = -k_{2\nu} g_{\nu\mu}^{\perp P}, \quad g_{\mu\nu}^\perp = g_{\mu\nu} - \frac{P_\mu P_\nu}{s}. \quad (20)$$

The operator for $L = 0$ is a scalar (we write $X^{(0)}(k^\perp) = 1$), and the operator for $L = 1$ is a vector, $X_\mu^{(1)} = k_\mu^\perp$. The operators $X_{\mu_1 \dots \mu_L}^{(L)}$ for $L \geq 1$ can be written in the form of a recurrency relation:

$$\begin{aligned} X_{\mu_1 \dots \mu_L}^{(L)}(k^\perp) &= k_\alpha^\perp Z_{\mu_1 \dots \mu_L}^\alpha(k^\perp) \equiv k_\alpha^\perp Z_{\mu_1 \dots \mu_L, \alpha}(k^\perp), \\ Z_{\mu_1 \dots \mu_L}^\alpha(k^\perp) &\equiv Z_{\mu_1 \dots \mu_L, \alpha}(k^\perp) = \frac{2L-1}{L^2} \left(\sum_{i=1}^L X_{\mu_1 \dots \mu_{i-1} \mu_{i+1} \dots \mu_L}^{(L-1)}(k^\perp) g_{\mu_i \alpha}^\perp - \right. \\ &\quad \left. - \frac{2}{2L-1} \sum_{\substack{i,j=1 \\ i < j}}^L g_{\mu_i \mu_j}^\perp X_{\mu_1 \dots \mu_{i-1} \mu_{i+1} \dots \mu_{j-1} \mu_{j+1} \dots \mu_L}^{(L-1)}(k^\perp) \right). \end{aligned} \quad (21)$$

We have a convolution equality $X_{\mu_1 \dots \mu_L}^{(L)}(k^\perp) k_{\mu_L}^\perp = k_\perp^2 X_{\mu_1 \dots \mu_{L-1}}^{(L-1)}(k^\perp)$, with $k_\perp^2 \equiv k_\mu^\perp k_\mu^\perp$, and the tracelessness property of $X_{\mu\mu\mu_3 \dots \mu_L}^{(L)} = 0$. On this basis, one can write down the normalization condition for orbital angular operators:

$$\int \frac{d\Omega}{4\pi} X_{\mu_1 \dots \mu_L}^{(L)}(k^\perp) X_{\mu_1 \dots \mu_L}^{(L)}(k^\perp) = \alpha_L k_\perp^{2L}, \quad \alpha_L = \prod_{l=1}^L \frac{2l-1}{l}, \quad (22)$$

where the integration is performed over spherical variables $\int d\Omega/(4\pi) = 1$.

Iterating Eq. (21), one obtains the following expression for the operator $X_{\mu_1 \dots \mu_L}^{(L)}$ at $L \geq 1$:

$$\begin{aligned} X_{\mu_1 \dots \mu_L}^{(L)}(k^\perp) &= \alpha_L \left[k_{\mu_1}^\perp k_{\mu_2}^\perp k_{\mu_3}^\perp k_{\mu_4}^\perp \dots k_{\mu_L}^\perp - \right. \\ &\quad \left. - \frac{k_\perp^2}{2L-1} \left(g_{\mu_1 \mu_2}^\perp k_{\mu_3}^\perp k_{\mu_4}^\perp \dots k_{\mu_L}^\perp + g_{\mu_1 \mu_3}^\perp k_{\mu_2}^\perp k_{\mu_4}^\perp \dots k_{\mu_L}^\perp + \dots \right) + \right. \\ &\quad \left. + \frac{k_\perp^4}{(2L-1)(2L-3)} \left(g_{\mu_1 \mu_2}^\perp g_{\mu_3 \mu_4}^\perp k_{\mu_5}^\perp k_{\mu_6}^\perp \dots k_{\mu_L}^\perp \right. \right. \\ &\quad \left. \left. + g_{\mu_1 \mu_2}^\perp g_{\mu_3 \mu_5}^\perp k_{\mu_4}^\perp k_{\mu_6}^\perp \dots k_{\mu_L}^\perp + \dots \right) + \dots \right]. \end{aligned} \quad (23)$$

For the projection operators, one has:

$$\begin{aligned} O &= 1, \quad O_\nu^\mu(\perp P) = g_{\mu\nu}^\perp, \\ O_{\nu_1 \nu_2}^{\mu_1 \mu_2}(\perp P) &= \frac{1}{2} \left(g_{\mu_1 \nu_1}^\perp g_{\mu_2 \nu_2}^\perp + g_{\mu_1 \nu_2}^\perp g_{\mu_2 \nu_1}^\perp - \frac{2}{3} g_{\mu_1 \mu_2}^\perp g_{\nu_1 \nu_2}^\perp \right). \end{aligned} \quad (24)$$

For higher states, the operator can be calculated using the recurrent expression:

$$O_{\nu_1 \dots \nu_L}^{\mu_1 \dots \mu_L}(\perp P) = \frac{1}{L^2} \left(\sum_{i,j=1}^L g_{\mu_i \nu_j}^\perp O_{\nu_1 \dots \nu_{j-1} \nu_{j+1} \dots \nu_L}^{\mu_1 \dots \mu_{i-1} \mu_{i+1} \dots \mu_L}(\perp P) - \frac{4}{(2L-1)(2L-3)} \times \sum_{\substack{i < j \\ k < m}}^L g_{\mu_i \mu_j}^\perp g_{\nu_k \nu_m}^\perp O_{\nu_1 \dots \nu_{k-1} \nu_{k+1} \dots \nu_{m-1} \nu_{m+1} \dots \nu_L}^{\mu_1 \dots \mu_{i-1} \mu_{i+1} \dots \mu_{j-1} \mu_{j+1} \dots \mu_L}(\perp P) \right). \quad (25)$$

The projection operators obey the relations:

$$\begin{aligned} O_{\nu_1 \dots \nu_L}^{\mu_1 \dots \mu_L}(\perp P) X_{\nu_1 \dots \nu_L}^{(L)}(k^\perp) &= X_{\mu_1 \dots \mu_L}^{(L)}(k^\perp), \\ O_{\nu_1 \dots \nu_L}^{\mu_1 \dots \mu_L}(\perp P) k_{\nu_1} k_{\nu_2} \dots k_{\nu_L} &= \frac{1}{\alpha_L} X_{\mu_1 \dots \mu_L}^{(L)}(k^\perp). \end{aligned} \quad (26)$$

Hence, the product of the two $X^L(k_\perp)$ operators results in the Legendre polynomials as follows:

$$X_{\mu_1 \dots \mu_L}^{(L)}(k_1^\perp) (-1)^L O_{\nu_1 \dots \nu_L}^{\mu_1 \dots \mu_L}(\perp P) X_{\nu_1 \dots \nu_L}^{(L)}(k_2^\perp) = \alpha_L \left(\sqrt{-k_1^{\perp 2}} \sqrt{-k_2^{\perp 2}} \right)^L P_L(z), \quad (27)$$

where $z \equiv (-k_{1\nu}^\perp k_{2\nu}^\perp) / (\sqrt{-k_1^{\perp 2}} \sqrt{-k_2^{\perp 2}})$.

Appendix B: Projection operators for baryon resonance states with arbitrary J .

The wave function of a resonance state with spin $J = n + 1/2$, momentum p and the effective mass term \sqrt{s} is given by a tensor four-spinor $\psi_{\mu_1 \dots \mu_n}$. It satisfies the constraints

$$(\hat{p} - \sqrt{s})\psi_{\mu_1 \dots \mu_n} = 0, \quad p_{\mu_i} \psi_{\mu_1 \dots \mu_n} = 0, \quad \gamma_{\mu_i} \psi_{\mu_1 \dots \mu_n} = 0, \quad (28)$$

and the symmetry properties

$$\begin{aligned} \psi_{\mu_1 \dots \mu_i \dots \mu_j \dots \mu_n} &= \psi_{\mu_1 \dots \mu_j \dots \mu_i \dots \mu_n}, \\ g_{\mu_i \mu_j} \psi_{\mu_1 \dots \mu_i \dots \mu_j \dots \mu_n} &= g_{\mu_i \mu_j}^\perp \psi_{\mu_1 \dots \mu_i \dots \mu_j \dots \mu_n} = 0. \end{aligned} \quad (29)$$

Conditions (28), (29) define the structure of the denominator of the fermion propagator (the projection operator) which can be written in the following form:

$$\begin{aligned} F_{\nu_1 \dots \nu_n}^{\mu_1 \dots \mu_n}(p) &= (-1)^n (\hat{p} + \sqrt{s}) \Phi_{\nu_1 \dots \nu_n}^{\mu_1 \dots \mu_n}(\perp p), \\ \mathfrak{P}_{\nu_1 \dots \nu_n}^{\mu_1 \dots \mu_n}(p) &= \frac{F_{\nu_1 \dots \nu_n}^{\mu_1 \dots \mu_n}(p)}{m^2 - s - i m \Gamma(s)} \end{aligned} \quad (30)$$

The operator $\Phi_{\nu_1 \dots \nu_n}^{\mu_1 \dots \mu_n}(\perp p)$ describes the tensor structure of the propagator. It is equal to 1 for a $(J = 1/2)$ -particle and is proportional to $g_{\mu\nu}^\perp - \gamma_\mu^\perp \gamma_\nu^\perp / 3$ for a particle with spin $J = 3/2$ (remind that $\gamma_\mu^\perp = g_{\mu\nu}^\perp \gamma_\nu$).

The conditions (29) are identical for fermion and boson projection operators and therefore the fermion projection operator can be written as:

$$\Phi_{\nu_1 \dots \nu_n}^{\mu_1 \dots \mu_n}(\perp p) = O_{\alpha_1 \dots \alpha_n}^{\mu_1 \dots \mu_n}(\perp p) \phi_{\beta_1 \dots \beta_n}^{\alpha_1 \dots \alpha_n}(\perp p) O_{\nu_1 \dots \nu_n}^{\beta_1 \dots \beta_n}(\perp p). \quad (31)$$

The operator $\phi_{\beta_1 \dots \beta_n}^{\alpha_1 \dots \alpha_n}(\perp p)$ can be expressed in a rather simple form since all symmetry and orthogonality conditions are imposed by O -operators. First, the ϕ -operator is constructed of metric tensors only, which act in the space of $\perp p$ and γ^\perp -matrices. Second, a construction like $\gamma_{\alpha_i}^\perp \gamma_{\alpha_j}^\perp = \frac{1}{2} g_{\alpha_i \alpha_j}^\perp + \sigma_{\alpha_i \alpha_j}^\perp$ (remind that here $\sigma_{\alpha_i \alpha_j}^\perp = \frac{1}{2}(\gamma_{\alpha_i}^\perp \gamma_{\alpha_j}^\perp - \gamma_{\alpha_j}^\perp \gamma_{\alpha_i}^\perp)$) gives zero if multiplied by an $O_{\alpha_1 \dots \alpha_n}^{\mu_1 \dots \mu_n}$ -operator: the first term is due to the traceless conditions and the second one to symmetry properties. The only structures which can then be constructed are $g_{\alpha_i \beta_j}^\perp$ and $\sigma_{\alpha_i \beta_j}^\perp$. Moreover, taking into account the symmetry properties of the O -operators, one can use any pair of indices from sets $\alpha_1 \dots \alpha_n$ and $\beta_1 \dots \beta_n$, for example, $\alpha_i \rightarrow \alpha_1$ and $\beta_j \rightarrow \beta_1$. Then

$$\phi_{\beta_1 \dots \beta_n}^{\alpha_1 \dots \alpha_n}(\perp p) = \frac{n+1}{2n+1} (g_{\alpha_1 \beta_1}^\perp - \frac{n}{n+1} \sigma_{\alpha_1 \beta_1}^\perp) \prod_{i=2}^n g_{\alpha_i \beta_i}^\perp. \quad (32)$$

Since $\Phi_{\nu_1 \dots \nu_n}^{\mu_1 \dots \mu_n}(\perp p)$ is determined by convolutions of O -operators, see Eq. (31), we can replace in (31)

$$\phi_{\beta_1 \dots \beta_n}^{\alpha_1 \dots \alpha_n}(\perp p) \rightarrow \phi_{\beta_1 \dots \beta_n}^{\alpha_1 \dots \alpha_n}(p) = \frac{n+1}{2n+1} (g_{\alpha_1 \beta_1} - \frac{n}{n+1} \sigma_{\alpha_1 \beta_1}) \prod_{i=2}^n g_{\alpha_i \beta_i}. \quad (33)$$

The coefficients in (33) are chosen to satisfy the constraints (28) and the convolution condition:

$$F_{\alpha_1 \dots \alpha_n}^{\mu_1 \dots \mu_n}(p) F_{\nu_1 \dots \nu_n}^{\alpha_1 \dots \alpha_n}(p) = (-1)^n F_{\nu_1 \dots \nu_n}^{\mu_1 \dots \mu_n}(p) \cdot 2\sqrt{s}. \quad (34)$$

The amplitude of photo-production (or, electro-production) of baryon resonances on nucleon target reads:

$$\mathfrak{A}_{\mu_1 \dots \mu_n}(p) = \frac{F_{\alpha_1 \dots \alpha_n}^{\mu_1 \dots \mu_n}(p)}{m^2 - s - i m \Gamma(s)} \mathfrak{G}_{\alpha_1 \dots \alpha_n}^{(a\pm)}(k_\perp), \quad (35)$$

remind, $a = 1, 2, 3$.

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